M. Sc. Sem I Paper 1st

ABSTRACT ALGEBRA

SOLVABLE GROUP

A group G is called solvable group if G has a series of subgroups

$$\{e\} = H_0 \subset H_1 \subset H_2 \subset \cdots \subset H_k = G$$

where, for each $0 \leq i < k$, H_i is normal in H_{i+1} and H_{i+1}/H_i is abelian.

In case, each H_i is a normal subgroup of H_{i+1} then the above series is known as the <u>sub-normal series</u> for G.

The quotient group H_{i+1}/H_i are called <u>factor groups</u> of the <u>subnormal series</u> for G.

Further, if each H_i is a normal subgroup of G then the above series is called <u>normal series</u> for G.

Theorem Every abelian group is solvable.

Soln.

Let G be an abelian group.

Let $H_0 = \{e\}$ and $H_K = G$.

$\Rightarrow \because \{e\}$ is a normal subgroup of G,

Since for any element a of G

$a^{-1}ea = a^{-1}ae = ee = e \in \{e\} = H_0$

Also, G is abelian, and we know that every quotient group of an abelian group is abelian.

$\Rightarrow$ the quotient group $G/\{e\}$ i.e. H_K/H_0 is also belian.

Thus $\{e\} = H_0 \subset H_K = G$ where $\{e\}$ is normal in G and $G/\{e\}$ is abelian.

By definition of solvable group, G is solvable.